"It isn't that they can't see the solution.
It is that they can't see the problem."

G. K. Chesteron, writer, 1874-1936

CONTACT INFORMATION

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WHY SHOULD YOU BE INTERESTED IN CALCULUS?

From Thomas & Finney's Text:

Economists use calculus to forecast global trends. Oceanographers use calculus to formulate theories about ocean currents and meteorologists use it to describe the flow of air in the upper atmosphere. Biologists use calculus to forecast population size and to describe the way predators like foxes interact with their prey. Medical researchers use calculus to design ultrasound and x-ray equipment for scanning the internal organs of the body. Space scientists use calculus to design rockets and explore distant planets. Psychologists use calculus to understand optical illusions in visual perception. Physicists use calculus to design inertial navigation systems and to study the nature of time and the universe. Hydraulic engineers use calculus to find safe closure patterns for valves in pipelines. Electrical engineers use it to design stroboscopic flash equipment and to solve the differential equations that describe current flow in computers. Sports equipment manufactures use calculus to design tennis rackets and baseball bats. Stock market analysts use calculus to predict prices and assess interest rate risk. Physiologists use calculus to describe electrical impulses in neurons in the human nervous system. Drug companies use calculus to determine profitable inventory levels and timber companies use it to decide the most profitable time to harvest trees. The list is practically endless, for almost every professional field today uses calculus in some way.

Mrs. Scheltz

BOSTON COLLEGIATE CHARTER SCHOOL

COURSE OVERVIEW

This course will have a heavy emphasis on conceptual understanding of differential and integral calculus. While it will be necessary for students to master the foundational skills and techniques, students must be able to explain the 'why' and the 'how' of the major ideas. Students will analyze ideas graphically, numerically, analytically, and verbally in order to construct deep understandings of the concepts.

Calculus I is based on three main concepts: limits (approaching a value), derivatives (rate of change), and integration (accumulation).

There will be daily quizzes so I know how well you're staying on top of the material, since the pace will be very fast. Unit exams are all divided into calculator and non-calculator sections.

This course will be aligned to the Advanced Placement Calculus AB test (there also exists a Calculus BC test that covers traditional Calc I and II). You are required to take the test at 8 am on Wednesday, May 9th. You are also required to go to three Saturdays of AP prep that will take place at high schools around Boston. Those Saturdays are:

December 17, 2011 March 10, 2012 April 28, 2012

TOPIC OVERVIEW

First Semester

Unit 1	Limits
Unit 2	Derivatives
Unit 3	Applications of Derivatives
Unit 4	Integration

Second Semester

Unit 5	Logarithms and Exponentials
Unit 6	Applications of Integration: Different Functions
Unit 7	Applications of Integration: Volume
Unit 8	Review for AP Exam

GRADING POLICY (PER QUARTER)

40%	Tests
30%	Quizzes (primarily Do Now quizzes)
15%	Homework
15%	Classwork

YEARLY GRADE BREAKDOWN

Q1	Q2	Mid-term	Q3	Q4	Final
23%	23%	7%	23%	11%	13%

MATERIALS/SUPPLIES

- Larson, Ron, Robert P. Hostetler, and Bruce H. Edwards. Calculus with Analytic Geometry. 8th ed. Boston: Houghton Mifflin Company, 2006.
- Publisher's website has worked out solutions to odd problems at www.CalcChat.com
- Binder with loose leaf paper
  OR
  Math folder plus a large spiral notebook for math only
- Pencils, erasers
- At least a TI-83 graphing calculator, no more than a TI-89

You are expected to bring all your materials to class every day. Failure to do so will result in demerits.

Supplementary Print Resources

Lifshitz, Maxine. Amsco's AP Calculus AB/BC: Preparing for the Advanced Placement Exams (Amsco's Ap). New York City: Amsco School Publisher Inc, 2004. **students may keep

Best, George, Stephen Carter, and Douglas Crabtree. Calculus: Concepts and Calculators. 2nd ed. Andover: Venture.

Finney, Ross L., Franklin D. Demana, Bert K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic. 3rd ed. Boston: Prentice Hall, 2007.

Hockett, Shirley O., and David Bock. Barron's AP Calculus 2008. 9th ed. Hauppauge: Barron's Educational Series, 2007.

HOMEWORK POLICY:

No late homework will be accepted. If a student is absent, he/she must turn in the homework the day he/she returns to school or within one day of returning and will receive full credit. The 12th Grade Absence and Late Submission Policy applies to this class.

ABSENT POLICY

Students are strongly encouraged not to be absent, but if you must, you will be responsible for turning in the assignment **due** the day they missed on the day they return. The homework **assigned** on the day they missed will be due the following day unless otherwise discussed. Students are responsible for seeing the teacher for missed assignments on the day of their return. If a student misses a test or a quiz, they are responsible for making up the missed quiz or test within a reasonable amount of time. They are still responsible for seeing me the day they return. Please refer to the 12th grade policy for all guidelines.

COURSE SEQUENCE

Unit 1 (Chapter 1) Limits and Their Properties
- Limits – graphically and numerically
- Evaluating limits analytically
- Continuity and one-sided limits
- Infinite limits

Unit 2 (Chapter 2) Differentiation
- The Tangent Line
- Basic differentiation rules
- Product and quotient rules
- Higher order derivatives
- The Chain Rule
- Implicit differentiation
- Related rates

Unit 3 (Chapter 3.1 - 3.6) Applications of Differentiation
- Extrema on an interval
- Rolle's Theorem
- Mean Value Theorem
- Increasing and decreasing functions
- Concavity
- Limits at infinity
- Curve sketching

Unit 4 (Chapter 3.7 – 3.9 ; 6.1 - 6.3 ; 8.1 - 8.2): Applications of Differentiation
- Optimization problems
- Newton's Method
- Differentials
- Differential equations: growth and decay
- Separation of variables and the Logistic Equation
- Projects (See Appendix 1)

Unit 5 (Chapter 4): Integration

- Antiderivatives and indefinite integration
- Area approximations
- Riemann sums and definite integrals
- The Fundamental Theorem of Calculus
- Integration by substitution
- Numerical integration

Midterm Exam: *Week of January 18th*

Unit 6 (Chapter. 5.1 - 5.7): Transcendental Functions

- Inverse Functions
- Differentiation and integration of the Natural log, Exponential Functions, and Inverse Trig Functions
- Bases other than *e*

Unit 7 (Chapter 7.1 - 7.3; 6.1): Applications of Integration

- Area of a region between two curves
- Slope Fields & Euler's Method
- Volume: The Disk & Shell Methods

Unit 8: Review for the exam

- We will use AMSCO's worksbook and take weekly quizzes on the problems done that week

Final Exam: *Week of April 30th*

WEBSITES

www.CalcChat.com

www.khanacademy.com

www.youtube.com (seriously!)

Topic Outline

This topic outline is intended to indicate the scope of the course, but it is not necessarily the order in which the topics need to be taught. Teachers may find that topics are best taught in different orders. Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and Limits

II. Derivatives

III. Integrals

I. Functions, Graphs, and Limits

A. Analysis of Graphs

With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

B. Limits of Functions (incl. one-sided limits)

- An intuitive understanding of the limiting process.
- Calculating limits using algebra.
- Estimating limits from graphs or tables of data.

C. Asymptotic and Unbounded Behavior

- Understanding asymptotes in terms of graphical behavior.
- Describing asymptotic behavior in terms of limits involving infinity.
- Comparing relative magnitudes of functions and their rates of change. (For example, contrasting exponential growth, polynomial growth, and logarithmic growth.)

D. Continuity as a Property of Functions

- An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits.
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

II. Derivatives

A. Concept of the Derivative

- Derivative presented graphically, numerically, and analytically.
- Derivative interpreted as an instantaneous rate of change.
- Derivative defined as the limit of the difference quotient.
- Relationship between differentiability and continuity.

B. Derivative at a Point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation.
- Instantaneous rate of change as the limit of average rate of change.
- Approximate rate of change from graphs and tables of values.

C. Derivative as a Function

- Corresponding characteristics of graphs of *f* and *f* '.
- Relationship between the increasing and decreasing behavior of *f* and the sign of *f* '.
- The Mean Value Theorem and its geometric consequences.
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

D. Second Derivatives

- Corresponding characteristics of the graphs of *f*, *f* ', and *f* ".
- Relationship between the concavity of *f* and the sign of *f* ".
- Points of inflection as places where concavity changes.

E. Applications of Derivatives

- Analysis of curves, including the notions of monotonicity and concavity.
- Optimization, both absolute (global) and relative (local) extrema.
- Modeling rates of change, including related rates problems.
- Use of implicit differentiation to find the derivative of an inverse function.
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

F. Computation of Derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- Basic rules for the derivative of sums, products, and quotients of functions.
- Chain rule and implicit differentiation.

III. Integrals

A. Interpretations and Properties of Definite Integrals

- Definite integral as a limit of Riemann sums.
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)dx = f(b) - f(a).$$

- Basic properties of definite integrals. (Examples include additivity and linearity.)

B. Applications of Integrals
Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the integral of a rate of change to give accumulated change or using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, and the distance traveled by a particle along a line.

C. Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals.
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

D. Techniques of Antidifferentiation

- Antiderivatives following directly from derivatives of basic functions.
- Antiderivatives by substitution of variables (including change of limits for definite integrals).

E. Applications of Antidifferentiation

- Finding specific antiderivatives using initial conditions, including applications to motion along a line.
- Solving separable differential equations and using them in modeling. In particular, studying the equation $y' = ky$ and exponential growth.

F. Numerical Approximations to Definite Integrals
Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

BOSTON COLLEGIATE CHARTER SCHOOL

LaDonne's syllabus

AP Calculus Syllabus

Course Overview

This course is a college-level course that can lead to college credit through successful performance on the AP exam. Through hard work this course will prepare students to take higher-level math courses in college. The basis of Calculus is the study of functions, specifically limits, derivatives, and integrals of functions. This course will examine each of these topics analytically, graphically, numerically, and verbally. As these different topics are explored, the class will seek to understand the meaning, significance and connection behind the skills and concepts learned so that they may be applied in a variety of situations and not just viewed as a set of isolated skills.

Calculators

Each student is provided with a TI-83 Plus calculator in September that they are expected to bring to class every day for use exploring concepts and solving problems. As the teacher I present any calculator work to the class through overhead projection of a TI-84 calculator.

Classwork and Assessments

Most classwork is done in small group exploration of topics. Selected activities have been listed and explained in-depth for each unit. These are representative of the type of activities students do in that unit, but do not represent a comprehensive list. Unit assessments are comprised almost entirely of previously released AP exam questions. In addition to regular classwork, beginning in December instruction is provided every other Saturday at the school for four hours. This time is used to take full-length practice AP exams and explore topics more in depth.

Primary Textbook

Larson, Ron, Robert P. Hostetler, Bruce H. Edwards, David E. Heyd. *Calculus of a Single Variable*. 7th ed. Boston: Houghton Mifflin, 2002.

Course Outline and Activities that address each of the following curricular requirements:

C2-The course teaches all topics associated with Functions, Graphs, and Limits; Derivatives; and Integrals as delineated in the Calculus AB Topic Outline in the *AP Calculus Course Description.*

C3-The course provides students with the opportunity to work with functions represented in a variety of ways—graphically, numerically, analytically, and verbally—and emphasizes the connections among these representations.

C4-The course teaches students how to communicate mathematics and explain solutions to problems both

C5- The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions.

Prerequisite Skills – 1 week

Students complete a review packet ove[...]nial, rational, exponential, logarithmic, trigo[...]ewise functions including their equations, how to evaluate various types of functions and recognize and sketch their graphs. They also review specific characteristics of various families of functions including domain, range, symmetry, zeros, intercepts, slope, and various transformations.

- Student Activity 1: Rule of 4 is introduced through Graphic, Numeric, Analytic, Words (GNAW) activity. **[C3] [C4]**

5-Day Walkthrough of Calculus – 1 week

- Student Activity 2: "Calculus: Concepts and Applications Instructor's Resource" by Curriculum Press
  Students complete five activities, one each day that introduces basic exploration of the three foundational concepts of Calculus: limits, derivatives, and integrals. **[C2] [C3] [C5]**

Limits and Continuity – 2 weeks

- Explore Limits Numerically using Calculator Table of Values
  - Introduce Limit notation
- Explore Limits Graphically using Calculator with window steps of .01 to illustrate holes—emphasize that what is happening at x=a is not particularly important, but can give insight into the limit
- Explore Limits Analytically using direct substitution, dividing, and rationalizing techniques
- One-sided limits and notation
- Infinite Behavior—explored through tables and graphs and then analytically
  - Asymptotic behavior
  - End behavior
- Continuity-students identify which graphs are continuous and then based upon different graphic representations of continuous and discontinuous functions determine the conditions for continuity
- Student Activity 3: Students compile limit skills through a project where they compile graphic and analytic representations of various functions that may be encountered on “Mt. Limit” journey that each illustrate different types of occurrences when evaluating limits and include a solution key for the “tools” available to solve that particular occurrence. Students then each present one assigned portion of the final project to the class. **[C2] [C3] [C4]**

Derivatives – 5 weeks

- Limit Definition of the Derivative
- Differentiability and Continuity

- Interpretations of the derivative: slope of a tangent line, rate of change, velocity and acceleration
  Student Activity 4: Students rotate through three stations each with the derivative of a function modeling a real-world problem where the derivative is interpreted as the slope of the graph of the function, the numeric rate of change of two quantities and particular examples involving velocity and acceleration. Two of the stations involve using the CBL and students must write-up their results and observations to turn in. **[C2] [C3] [C4] [C5]**
- Differentiation Rules/Shortcuts
  - Power Rule
  - Constant Multiple Rule
  - Sum/Difference Rule
  - Product Rule
  - Quotient Rule
  - Trig Function Derivatives
  - Composite Functions/Chain Rule
  - Implicit Differentiation
  - **Exponential and Logarithmic Rules**
- Related Rates
  Student Activity 5: Students complete a variety of simulations in small groups to discover all of the things that are changing and how those changing values are related to each other so that as one changes, the other does as well, but at a different rate. Students then present their particular simulation and results to the rest of the class. **[C2] [C4]**

Applications of the Derivative – 4 weeks

- Extrema (relative and absolute)
- **Rolle's Theorem**
- **Mean Value Theorem**
- Increasing/Decreasing
  - Critical Numbers and 1st Derivative Test for Relative Extrema
- Concavity and Points of Inflection
  - 2nd Derivative Test for Relative Extrema
- Relationship between f(x), f'(x), and f''(x)
  - Student Activity 6: Matching graphs with their derivatives **[C2] [C3] [C4]**
- Optimization
  - Student Activity 7: Optimizing Packaging **[C2] [C3] [C5]**

Antiderivatives and Slope Fields – 2 weeks

- Antiderivatives and Basic Antiderivative Rules
  - Power Rule
  - Constant Multiple Rule
  - Sum/Difference Rule
  - Trig Antiderivatives
- Particular solutions

- **Slope fields**
- **Differential equations**
  - **separation of variables**
  - **growth and decay**

Definite Integrals – 8 weeks

- Riemann Sums (right, left, midpoint)
- **Trapezoidal Rule**
- Limit definition of the definite integral
- The 1st part of the Fundamental Theorem of Calculus
  - Student Activity 8: "Functions Defined by a Definite Integral" by Mark Howell **[C2] [C3]**
  - area between two curves
  - Area vs. Net Change
    - Student Activity 9: "From Riemann Sums to Net Change" by Ray Cannon **[C2] [C3] [C5]**
- **Average Value Theorem**
- Volume
- Student activity 10: Finding the volume of donuts and donut holes **[C2] [C4]**
  - disk method
  - shell method
  - solids with known cross sections

Indefinite and Definite Integrals as functions – 2 weeks

- **The 2nd part of the Fundamental Theorem of Calculus and properties of Definite Integrals**
  - **Student Activity 11: "The Integral Function" by Benita Albert [C2]**
- **Integration by Substitution**

Student Activities

Activity 1: GNAW Activity where students explore various concepts including finding zeros of quadratic equations and solving a system of equations using all four methods. Students are required to write verbal explanations of solutions and present solutions to the class explaining the connections between the GNAW results and explain that certain information or types of problems lend themselves to one of the four methods and why graphs and numeric tables are not sufficient to prove an idea. **[C3] [C4]**

Activity 2: 5 Day Walkthrough
Day 1: Instantaneous Rate of Change--Students explore the movement of an opening and closing door hinge in terms of the angle measure the door hinge makes as time passes. They analyze the movement graphically, numerically and analytically to discover the rate the angle is changing varies as time varies.

Day 2: Exploring the slope of various graphs and where they are increasing, decreasing, or constant and how steep the slope is at particular points.
Day 3: Definite Integrals as area under a curve representing distance and volume. By analyzing velocity/time and area/width graphs students compute the area under a curve using rectangular approximation to explore meanings of these areas in terms of distance and volume.
Day 4: Definite Integrals as Trapezoidal approximations--Students use a velocity/time graph to estimate distance traveled by calculating area under the curve using different numbers of trapezoids.
Day 5: Introduction to Limits--Students analyze a rational function with a hole at a certain point graphically, numerically and analytically to determine the limit at that point with the hole.
[C2] [C3] [C5]

Activity 3: Mt. Limit Journey--examples of previous students' projects are shown in class. Each students is presented with one of the instances that they will present to the class
For this project you will draw a graph that contains the general graphic possibilities when finding a limit. There are seven possibilities that have been discussed in Chapter 1 including: continuous point, hole, oscillating, jump, infinite behavior (two-sided positive infinity, two-sided negative infinity, and different one-sided behavior approaching infinity and negative infinity).
Your drawing should be on the paper provided and include:

- An instance for each of the seven possible graphic results when finding a limit.
- Labels on the x-axis for the value x is approaching at each of the seven instances. You may use numbers or letters so long as they are distinct.
- Labels on the f(x)-axis for the limit values. Again, you may use numbers or letters so long as they are distinct.

Your solution key should include the following, in the order given:

- Two specific examples of functions that exhibit this type of behavior.

For the following items use a generic f(x) notation in the limits indicating that this behavior is representative of various types of functions of which you just gave two specific examples.

- What is occurring at the given point on the graph?
- The proper limit notation for your function at each respective point
- The proper notation and solution to the one-sided limit from the left
- The proper notation and solution to the one-sided limit from the right
- The solution to the two-sided limit
- All the possible strategies available for finding the solution to the limit (GNAW and if analytic, the specific techniques (direct substitution, rationalization, dividing/factoring))
- Is the function continuous or discontinuous, explain your answer

[C2] [C3] [C4]

Activity 4:

Distance Match: Students attempt to match distance graphs using the CBL and motion detectors. They analyze each attempt and write why their matches were not accurate until they have a good match. They then sketch velocity and acceleration graphs that would correspond to their matching distance graph and write piecewise equations for all of their graphs. Finally they determine the relationship between each graph as slope, rate of change, and derivatives.
Ball Toss: Students toss a ball and measure its position using a motion detector. They complete a lab similar to the distance match, but this time use the collected time and position data and the regression capability of the graphing calculator to determine the equation for the position of the ball and the numeric, analytic, and graphic representations of the velocity and acceleration of the ball.
Currency Exchange Rates: Students analyze the exchange rate of a rupiah to a dollar over a given time period and using an algebraic model determine the average and instantaneous rates of exchange at intervals and particular time periods using algebraic and numeric techniques.
[C2] [C3] [C4] [C5]

Activity 5: Simulating Related Rates--Students are divided into small groups. One group is blowing up balloons and measuring the change in radius and time at periodic intervals. They are then determining the change in volume at a particular time given the rate of change of the radius at that time. Another group is dripping water out of the bottom of a conical cup and measuring the change in height and proportional change in radius as time changes to determine these rates of change and their relationship to the rate of change of the volume. Another group is measuring the distance between two students position in relation to their shared starting point as one walks west and another walks south and measuring their respective rates of change to determine the rate of change of the distance between them at a given time. Another group is measuring the rate of change of water being poured into a conical cup and determining the rate of change of the height and radius (to discover the radius isn't changing) as time changes and using this information to determine the rate of change of the volume. Students complete their group write-ups and present their results to the rest of the class.
[C2] [C4]

Activity 6: f(x), f'(x), f''(x)
How are your grandmother, mother, and you related? We discuss this relationship as a beginning point for making the connection between the relationship between f(x) and f'(x) and f'(x) and f''(x).
How are functions and their derivatives related? Use the similarities and differences between the relationships between generations to describe the relationships between functions, their derivatives. Specifically use the characteristics of positive, negative, zero, increasing, decreasing, constant, concave up and concave down to describe how looking at one function can tell you the characteristics of its derivatives or antiderivatives. After completing this writing task each student is given one or two graphs. They must use their writing to determine what the derivative and antiderivative of their graph would look like using some key characteristics and then must find their derivative or antiderivative graph amongst the graphs their classmates have. Once they

have found their match, in pairs, they explain to the class what characteristics on the graphs lead them to their match. **[C2] [C3] [C4]**

Activity 7: Optimizing Packaging materials
Students bring in rectangular and cylindrical packages from home. We have a brief discussion in at the beginning of class about why certain things might be packaged the way that they are. They then use the boxes and cans brought into class to complete the following activity:

- Take a box and determine the perimeter of its front. Is there a way to use the same number of inches in the perimeter, but increase or decrease its area? Create a table with the possible dimensions that maintain the perimeter of the front of the box.
- Does it appear that there are certain dimensions that maximize or minimize the area of the front of the box?
- Now take one cylinder and determine its volume in cubic inches. Write an equation for the volume of a cylinder with that volume value. Then create a table of values for all of the possible dimensions of a cylinder with that volume.
- Calculate the surface area for each of the dimensions in the table above. Is there a certain dimension that maintains the volume, but minimizes the surface area?

Students use table and plotting features of graphing calculators to generate various coordinates and sketch the graphs of the length/width graph to see graphically the maximum that they had discovered numerically. This activity was adapted from an activity in the textbook. **[C2] [C3] [C5]**

Activity 8: Mark Howell's "Functions Defined by a Definite Integral"
Students explore the relationship between area under a curve and the definite integral by calculating basic areas under a constant function and discovering each of the area values corresponds to a point on the antiderivative function. They also explore how the definite integral changes as the upper bound changes and through these changes discover the Fundamental Theorem of Calculus F(b)-F(a). This activity begins exploration of the 2nd part of the Fundamental Theorem of Calculus, which will be explored further in Activity 11. **[C2] [C3]**

Activity 9: "From Riemann Sums to Net Change" by Ray Cannon
Students explore the difference between a definite integral set up to yield area and a definite integral set up to yield net change. They do this through a position vs. distance traveled scenario of a rock being thrown into the air. Students set up and evaluate definite integrals with a velocity that is positive and negative over certain intervals so that the integral sometimes yield position (or net change) and then determine how to change the definite integral so it would yield distance traveled (or area). Students use graphing calculators to perform some of the operations and to sketch the graph of the situation. **[C2] [C3] [C5]**

Activity 10: Volume of donuts and donut holes
Students are shown a graph of a cross section of a donut graphically and told that it is rotated around an axis to produce a 3D shape that looks like the donut. Students are

commissioned with the task of determining the volume of the resulting donut or donut hole. They are given plastic knives or dental floss and a ruler as their only tools. Students must explore the difference between disk and washer methods because the donuts have space between the curve and axis of revolution and the donut holes do not. Students must discuss their strategies for determining the volume and write-up their final strategy. **[C2] [C4]**

Activity 11: "The Integral Function" by Benita Albert
Students experiment with the idea that a definite integral could define a function, not just represent and area or net change. They use graphic representations to interpret the meaning of the definite integral as the bounds change and use the similarities and differences between the various functions defined as the upper limit changes to reflect on properties of the definite integral and its relationship to the integrand. **[C2]**